HEAT FLUX THROUGH A PHASE INTERFACE IN SUPERFLUID HELIUM

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The heat flux between phases is calculated as a function of the degree of submersion and the temperature of the bath on the basis of solution of the hydrodynamic problem for superfluid helium.

The possibility of utilizing superfluid helium as a coolant in cryogenic systems (see the surveys [1, 2], say) has been discussed extensively recently. Heat transfer with this medium is realized quite uniquely in the film nucleation mode. One of the classical papers on He II boiling is the paper by W. Rivers and P. MacFadden [3], in which helium boiling on a cylinder and vertical plate was studied by using a boundary-layer approximation in a vapor film (or a He I film). The role of the superfluid helium was here reduced to the fact that it regulated the heat flux, transferred from the phase interface to the free surface. This quantity, denoted as $q_B(T_B, H)$ in [3], was the parameter of Rivers and MacFadden's theory and not calculated.

The quantity q_B was computed from the viewpoint of a nonequilibrium molecular theory in [4]. The main idea of the paper was to take into account that the interphasal heat flux produces an additional pressure due to the nonequilibrium of the process. However, even here a certain assumption was utilized about the proportionality of this heat flux to the temperature drop in helium: $\Delta = T_\delta - T_B$. The proportionality factor is here an adjustment parameter determined from comparison with experiment. Therefore, the question of a theoretical set-up of the relationship between the quantities q_B, H, and T_B remains open. The problem of the temperature T and pressure p distributions and the velocity of the normal component v_n in the superfluid helium layer bounding its vapor or solid surface (from below) is solved in the paper presented. On the basis of the solution of this problem, the interphasal heat flux q_B is solved as a function of the free surface temperature T_B and the degree of submersion H.

1. Case of Small Heat Fluxes

Let us consider an infinite horizontal plate submerged a depth H in superfluid helium, at which the heat flux $q > q_{cr}$ is liberated, i.e., a film boiling mode is realized. The temperature in the He II surface is T_B and the saturation pressure p_B corresponds. Since the thickness of the vapor film δ is small compared with H, we shall consider the phase interface to be at the depth H also. We take this surface as the origin (x is here directed upward).

For the sequel, it is useful to recall the discussion of Rivers and MacFadden [3] resulting in the deduction about the existence of a flux q_B given uniquely by the quantities H and T_B. We consider the phase diagram of helium (Fig. 1). Let the point A correspond to the free helium surface. The assumption was made in [3] that there is no temperature drop in the helium volume, while the pressure varies according to the hydrostatic law $p(x) = p_B + \rho g(H - x)$. Consequently, the state of the helium near the heating element is portrayed by the point B on the segment of the isotherm AB. Since the lower boundary is assumed in thermodynamic equilibrium with the vapor, then its temperature is fixed and determined by the saturation curve (the point C). Rivers and MacFadden [3] therefore arrived at the deduction about the existence of a temperature jump on the lower boundary of the helium volume, which should correspond, in their opinion, to a strictly definite heat flux density q_B. It is evident from the discussion that this density is determined just by the degree of submersion H and the temperature T_B. It will be shown below that this reasoning is not totally accurate.

As is known (see [2], say), quantized vortex filaments develop in helium flows with velocities exceeding 10^{-1} cm/sec (which corresponds to heat fluxes $q \approx 10^{-3}$ W/cm²). Since heat

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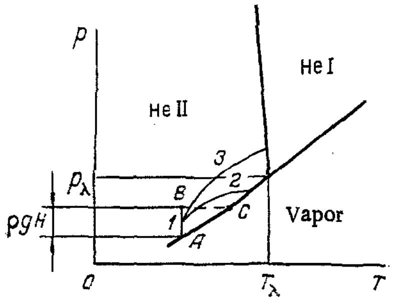

Fig. 1

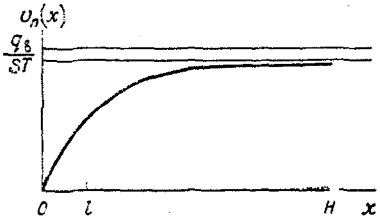

Fig. 2

Fig. 1. Helium phase diagram: 1-3) thermodynamic state of helium with the heat flux q_B present; curve 1 is taken from [3], while curves 2 and 3 correspond to the results in this paper.

Fig. 2. Schematic diaplay of the dependence of the normal velocity component v_n on the coordinate x.

fluxes in helium boiling are known to exceed this quantity, it is then necessary to use the equations of helium motion that take account of the existence of such vortex filaments. Such equations were obtained by Lebedev and Nemirovskii in [5]. In order not to complicate the text excessively, we do not write these complex equations down, especially for the case which is to be investigated when they simplify substantially. We discuss only the conditions resulting in simplification of the equations, referring here to the original paper [5].

First, we are interested in the stationary case ($\partial/\partial t = 0$). This permits expressing the quantity L in [5] (the density of the vortex ball) in terms of the normal and superfluid velocity components $L = \alpha^2/\beta^2(v_n - v_s)^2$. Furthermore, since there is no total mass flux, then $j = \rho_s \mathbf{v}_s + \rho_n \mathbf{v}_n = 0$. This permits elimination of one of the velocities $\mathbf{v}_s$, say, from the equations. For the case of small fluxes, the nonlinear terms of the equation can be omitted and their influence will be discussed below. Here, however, the nonlinear term which is substantially the Gorter–Mellink force (see [2]) must be retained in the equation for the superfluid velocity component since it governs the temperature drop in the helium bulk. Indeed, in the absence of this force an infinitesimal temperature gradient can cause enormous heat fluxes capable of producing compact vortex balls. The mutual friction force hinders infinite growth of the heat flux. Furthermore, although terms corresponding to ordinary dissipative effects are omitted in [5], they should be included because the influence of viscosity is substantial near the boundary x = 0. Consequently, we arrive at the following system of equations

$$\frac{\partial p}{\partial x} + \rho g = \frac{4}{3}\eta\frac{\partial^2 v_n}{\partial x^2} - \zeta_1\rho\frac{\partial^2 v_n}{\partial x} + \zeta_3\frac{\partial^2 v_n}{\partial x^2}, \tag{1}$$

$$-S\frac{\partial T}{\partial x} + \frac{\partial p}{\partial x} - \rho g = (-\zeta_3\rho^2 + \zeta_4\rho)\frac{\partial^2 v_n}{\partial x^2} + A(T)\rho_s\rho_n\left(\frac{\rho}{\rho_s}\right)^3 v_n^3, \tag{2}$$

$$S\frac{\partial v_n}{\partial x} - \frac{\varkappa}{T}\frac{\partial^2 T}{\partial x^2} = 0. \tag{3}$$

Let us clarify the meaning of (1)-(3).* Equation (1) is the conservation law for momentum flux. There are no nonlinear terms therein, but viscous terms are included in the momentum flux tensor [6]. Relationship (2) is the equation for the motion of the superfluid component in which the velocity v_s is replaced by $-\rho_n v_n/\rho_s$. Here the mutual friction forces, the viscous terms, and the gravitational term ρg are added. The last equation (3) is the entropy conservation law. The irreversible heat flux $\varkappa(dT/dx)$ is therein besides the equilibrium entropy flux Sv_n.

Relationships (1)-(3) are a closed system of equations for the quantities T, p, and v_n. This system must be supplemented by boundary conditions. We select the following:

$$T(0) = T_\delta,\quad p(0) = p_\delta,\quad T(H) = T_B,\quad p(H) = p_B, \tag{4}$$

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*Here and henceforth, $v_n(v_s)$ denotes the x components of the vectors $\mathbf{v}_n(\mathbf{v}_s)$.

where T_δ, p_δ, as well as T_B, p_B are interrelated by thermodynamic equilibrium conditions (they belong to the saturation curve).

The first integral of (3) has the form

$$Sv_n - \frac{\varkappa}{T}\frac{\partial T}{\partial x} = \text{const.} \tag{5}$$

The integration constant in the physical sense is q_B/T, since (5) describes the heat flux.

After evident manipulations, (1), (2), and (5) reduce to the form

$$\frac{S^2T}{\varkappa}v_n - \frac{Sq_b}{\varkappa} = \Lambda_T\frac{\partial^2 v_n}{\partial x^2} + A(T)\rho\rho_n\left(\frac{\rho}{\rho_s}\right)^3 v_n^3. \tag{6}$$

Here $\Lambda_T = \frac{4}{3}\eta - \zeta_1\rho - \zeta_4\rho + \zeta_2 + \rho^2\zeta_3$.

The solutions of (6) contain two constants, and integration of (5) to determine the temperature field adds still another. We have available just two constants T(0) and T(H). Therefore, there is some arbitrariness apparently in determining the temperature, velocity, and flux fields. In particular, $v_n(0)$ can be arbitrary. This means that any heat flux can correspond to a given temperature drop. It must be noted that conservation of the nonlinear terms also does not alter the situation being created. Therefore, the assumption of Rivers and MacFadden about the single-valued connection between the temperature drop and the heat flux does not seem to be confirmed.

The situation changes radically, however, when the quantum-mechanical nature of He II is taken into account. The fact is that the density of the superfluid component on the boundary of the system volume must vanish. This is a consequence of the quantum nature of superfluid helium according to which ρ_s is the modulus of a certain effective wave function that vanishes on the system boundary (see [7]). Since the total flux is $\rho_s v_s + \rho_n v_n = 0$, we obtain that v_n should also vanish on the boundary. We have therefore obtained the missing boundary condition $v_n(0) = 0$.

The additional condition $v_n(0) = 0$ reduces the arbitrariness in the formulation of the problem and imposes a single-valued relationship between the temperature drop and the heat flux q_B. Thus, the assumption of the existence of a definite q_B as a function of T_B and H is not as trivial as was represented earlier.

Equation (6) is solved analytically, permitting obtaining and investigating the solution of the problem posed in the most general form. We turn to dimensionless variables V and x' in (6), which are defined as follows:

$$v_n = V\frac{q_B}{ST}, \quad x' = x'\sqrt{\frac{\Lambda_T\varkappa}{S^2T}}.$$

The quantity $l = \left(\frac{\Lambda_T\varkappa}{S^2T}\right)^{1/2}$, with the physical meaning of the thickness of the near-boundary heat wave, was encountered earlier in the literature [7, 8]. Equation (6) in the variables V and x' takes the following form (we henceforth omit the prime):

$$V - 1 + A''V^3 = \frac{\partial^2 V}{\partial x^2}. \tag{7}$$

Here

$$A'' = -A(T)\rho\rho_n\left(\frac{\rho}{\rho_s}\right)^3\frac{\varkappa q_B^2}{S^4T^3}.$$

The boundary conditions are the following:

$$V(0) = 0, \quad V + V^3A''|_{x=H} = 1. \tag{8}$$

We solve (7). To do this, we make the substitution $\partial V/\partial x = Y$. We then have

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial V}{\partial x} = \frac{\partial Y}{\partial x}, \quad \frac{\partial Y}{\partial x} = \frac{\partial Y}{\partial V}\frac{\partial V}{\partial x} = Y\frac{\partial Y}{\partial V} = \frac{\partial Y^2/2}{\partial V}.$$

Substituting these relationships into (7), we obtain $\frac{\partial Y^2/2}{\partial V} = V - 1 + A''V^3$, from which

$$Y=\left(V^2-2V+\frac{2}{3}A''V^4+C_1\right)^{1/2}. \tag{9}$$

Integrating (9), we have that V(x) is expressed implicitly by using the elliptic integral

$$\int\limits_0^{V(x)}\frac{\partial V}{\sqrt{V^2-2V+\frac{2}{3}A''V^4+C_1}}=x, \tag{10}$$

which yields the desired distribution V(x). Furthermore, by using (5) and (1), the temperature T(x) and pressure p(x) distributions can be calculated. In the general case the left side of (10) can be found numerically. However, in a large range of heat flux and temperature quantities (far from T_λ), the term $A''V^4 \ll 1$ and the integral can be calculated by expanding the radical in series. Typical velocity, temperature, and pressure distribution patterns are displayed in Figs. 2-4.

The magnitude of the normal velocity component (Fig. 2) changes radically for small x, i.e., near the lower phase interface. For very small A" the velocity is $v_n \sim (\exp(x/l)-1)$. As x grows $v_n(x)$ approaches the asymptote almost equal to q_B/ST. The slight difference from this quantity results because a certain part of the heat flux is transferred irreversibly because of the temperature gradient. More accurately, at large distances from the lower phase boundary, the following will be valid:

$$q_{\text{в}}=STv_n-\varkappa\frac{\partial T}{\partial x},$$

which understandably agrees with boundary condition (8).

An abrupt change in the temperature curve (3) is observed near the helium lower boundary, which then smoothly approaches the asymptote defined by the equation

$$\frac{\partial T}{\partial x}=-A(T)\,\rho\rho_n\left(\frac{\rho}{\rho_s}\right)^3\frac{q_{\text{в}}^3}{S^4T^3}.$$

The total temperature drop is expressed as follows:

$$\Delta T=\int\limits_0^H\frac{\partial T}{\partial x}dx=\frac{q_{\text{в}}l}{\varkappa}+A(T)\,\rho\rho_n\cdot\left(\frac{\rho}{\rho_s}\right)^3\frac{q_{\text{в}}H}{S^4T^3}. \tag{11}$$

This formula is again valid for A" < 1. Therefore, the total temperature change consists of the known temperature jump [7, 8] and the drop caused by the mutual friction [2]. For $H = 10^2$ cm and T = 1.9°K the second term in (11) exceeds the first for $q_B = 10^{-1}$ W/cm^2.

The pressure distribution has the form displayed in Fig. 4. A steep change $p(x) \sim \exp(-x/l)$ is also observed here, which then becomes more shallow, approaches the asymptote, and satisfies the equation

$$\frac{\partial p}{\partial x}=-\rho g.$$

The total pressure change is

$$\Delta p=\int\limits_0^H\frac{\partial p}{\partial x}dx=\frac{\Lambda_p q_{\text{в}}}{STl}+\rho gH, \tag{12}$$

here $\Lambda_p=\frac{4}{3}\eta-\zeta_1\rho+\zeta_2$.

This last result is quite important. Insofar as the authors know, the question of the additional contribution (as compared with the hydrostatic) to the pressure has not been discussed in the literature. At the same time, this addition can be quite significant and even exceed the hydrostatic pressure noticeably. For example, for H = 10 cm, T = 1.9°K, and q_B = 1 W/cm^2 the first term in (12) is approximately 10^4 g/cm·sec^2, while $\rho gH = 1.4\cdot10^3$ g/cm·sec^2.

By analyzing the equation obtained, it can be said that the physical pattern of the process proposed by Rivers and MacFadden [3] is not accurate. If the thermodynamic state of He II in [3] was portrayed by a segment of the isotherm AB (see Fig. 1) of length ρgH, then certain other curves, for instance, curves 2 or 3, would correspond to our results. The total temperature ΔT and pressure Δp drops are calculated from (11) and (12). It is impor-

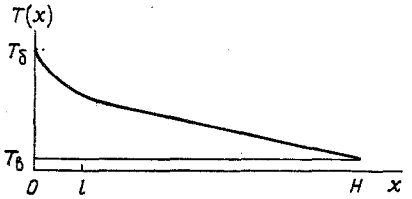

Fig. 3

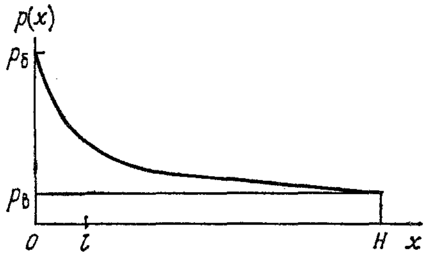

Fig. 4

Fig. 3. Temperature dependence on the coordinate x.

Fig. 4. Pressure dependence on the coordinate x.

tant to keep in mind here that, in contrast to the case of Rivers and MacFadden, the form of these curves depends on q_B and a situation can be encountered for which an increase in the flux will result in an additional contribution to the pressure Δp such that $p_B + \Delta p > p_\lambda$, and then He I rather than a vapor will be the boundary to the He II layer.

A formula relating q_B to the quantities T_B and H can be obtained from (11) and (12). In fact, if it is considered that the interphasal surfaces are in thermodynamic equilibrium, then

$$\Delta T = \left(\frac{\partial T}{\partial p}\right)_{\text{sat}} \Delta p.$$

Here $(\partial T/\partial p)_{\text{sat}}$ is the derivative to the equilibrium curve between the liquid and vapor. Since the quantities q_B, T_B, and H are parameters in these relations, then the deduction can be made that there is a single-valued connection between them. Simple computations result in the following formula for q_B as a function of T_B and H (in implicit form):

$$\frac{q_B l}{\varkappa} + A(T)\rho\rho_n\left(\frac{\rho}{\rho_s}\right)^3 \frac{q_B^3 H}{S^4 T^3} = \left(\frac{\partial T}{\partial p}\right)_{\text{sat}} \left[\frac{\Lambda_p q_B}{STl} + \rho g H\right], \tag{13}$$

which is, however, valid only for not very large temperature drops because, first, the linear solution is invalid in the case of large drops and, second, the relation $\Delta T = (\partial T/\partial p)_{\text{sat}}\Delta p$ is not true. For large Δp formulas (11) and (12) as well as tables must be utilized to determine the fluxes. There is a serious obstacle to computations of $q(T_B, H)$ by these relationships. The fact is that at this time there are no reliable data on the kinetic coefficients for He II which are in the quantities l and Λ_p; consequently, computations by the formulas presented will be only estimates in nature.

Let us note another circumstance. In substance, the solution of the problem is valid for the case of heat flux from a solid substrate. Indeed, up to the very last step (formula (13)) it was nowhere used that vapor is under the He II layer. In such a variant, (11) and (12) can be used to compute q_{cr}. Here, however, there is a complication, in principle, that ΔT and Δp are already not related by phase equilibrium conditions, but on the contrary, helium overheats near a solid surface. The overheating ΔT_{over} is determined by the surface material, the processing, etc.* Measurement or computation of ΔT_{over} is an independent problem. However, if a certain ΔT_{over} is given, then in this case the critical heat flux can be computed from the formulas presented by keeping in mind that the difference $\Delta T - \Delta T_{\text{over}}$ and the quantity Δp vary along the equilibrium curve.

In concluding this part, we discuss the following. If the quantity Δp calculated by means of (12) results in the inequality $p_B + \Delta p > p_\lambda$ (see Fig. 1, curve 3), then nonsuperfluid helium will be the boundary with He II. In this case, the examination presented in this paper is invalid. Indeed, without speaking about the fact that (1)-(3) are unsuitable in the region of the phase transition He II—He I we are simply not able to use the formulas obtained since many quantities are simply not defined at the transition point (are singular). In this case consideration of the problem should be performed specially.

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*As is known, in the theory of homogeneous nucleation there is no threshold value for ΔT_{over}. On the contrary, any arbitrarily small overheating produces a small but finite probability of vapor seed formation. However, if a graph of the vapor seed density is displayed as a function of ΔT, then it can be seen that this curve grows abruptly for certain ΔT. This condition can provisionally be chosen as the threshold value of ΔT_{over}.

2. Case of High Fluxes

In this case the problem presented should be considered with nonlinear terms taken into account. Simple calculated estimates show that certain of them exert no substantial influence on the results obtained up to fluxes equal to tens of watts per square centimeter. For instance, the nonlinear term in the momentum flux tensor $\rho_s v_{si} v_{sk} + \rho_n v_{ni} v_{nk}$, which yields a contribution to the pressure in the final answer, only changes this latter by percents. Analogously, the contribution from terms related to the vortices which have the form $\varepsilon_B L \delta_{ik}$ (see [5]) is also not large. The nonlinear terms resulting in a temperature change also yield a moderate change. Indeed, the temperature gradient characteristic for the nonlinear case can be estimated from the equation for the superfluid velocity v_s taken to second-order accuracy [6]:

$$\frac{\partial \mathbf{v}_s}{\partial t} + \nabla \left\{ \mu(p,\ T) + \frac{\mathbf{v}_s^2}{2} - \frac{\rho_n}{2\rho} (\mathbf{v}_n - \mathbf{v}_s^2) \right\} = 0. \tag{14}$$

The chemical potential depends only on the pressure and temperature but not on the relative velocity $v_n - v_s$ (see [6]):

$$d\mu = -\sigma dT + \frac{1}{\rho} dp,$$
$$\frac{\partial \mathbf{v}_s}{\partial t} + \frac{1}{\rho} \nabla p - \sigma \nabla T + \nabla \left[\frac{\mathbf{v}_s^2}{2} - \frac{\rho_n}{2\rho} (\mathbf{v}_n - \mathbf{v}_s)^2 \right] = 0. \tag{15}$$

Let us note that in the linear case there is the equality $\sigma \nabla T = (1/\rho) \nabla p$, i.e., the London relationships are satisfied. In the nonlinear case, the following estimate for the temperature gradient follows from (15):

$$|\nabla T| \simeq \frac{\rho_n}{\rho_s} \frac{1}{\sigma H} \frac{q_B^2}{S^2 T^2}. \tag{16}$$

For $T = 1.9$°K and $q_B = 1\ \mathrm{W/cm^2}$ we have $|\nabla T| \approx 10^{-5}$°K/cm, while because of the Gorter–Mellink force the temperature gradient for the same parameters equals $|\nabla T| \approx 10^{-3}$°K/cm. Therefore, the nonlinear terms described exert no strong influence on the temperature distribution. Nevertheless, there is still another reason for taking account of the nonlinear terms in the equations. Although the temperature gradients in the helium layer are not large, the total temperature drop because of the large distances can be quite significant. For instance, for $q_B = 3\ \mathrm{W/cm^2}$, $T = 1.9$°K, then $|\nabla T| = 10^{-2}$°K/cm, which yields one degree per meter. For such drops it is naturally impossible to consider the thermodynamic quantities constant; consequently, nonlinear terms appear. For example, the expression $\partial S v_n / \partial x$ should be as follows: $\frac{\partial S}{\partial T} \frac{\partial T}{\partial x} v_n + S \frac{\partial v_n}{\partial x}$, and not $S \frac{\partial v_n}{\partial x}$, as before.

Let us examine what would be the result of such corrections. First, the change in density can be neglected since the quantity $\partial \rho / \partial T$ is small in He II. The temperature jump near the boundary $\Delta T_b = q_B l / \varkappa$ is also not large (for $T = 1.9$°K and $q = 1\ \mathrm{W/cm^2}$ the quantity $\Delta T_b = 10^{-3}$°K, i.e., there will be no noticeable change in the parameters in the layer near the boundary). Outside this layer, the terms with the kinetic coefficients can be neglected in (1)-(3). Also $\partial p / \partial x = \rho g$ can be considered outside this layer since, as has already been said, the total density is unchanged. We finally have

$$\frac{\partial \mu}{\partial x} + \rho g = A(T) \rho_n \left(\frac{\rho}{\rho_s} \right)^3 v_n^3, \tag{17}$$

$$\frac{\partial S v_n}{\partial x} = 0. \tag{18}$$

Taking into account that $\partial \mu / \partial p = 1/\rho$, while $\partial p / \partial x = -\rho g$, we obtain

$$\frac{\partial \mu}{\partial T} \frac{\partial T}{\partial x} = A(T) \rho_n \left(\frac{\rho}{\rho_s} \right)^3 v_n^3, \tag{19}$$

$$T S v_n = q_B. \tag{20}$$

From (19) and (20) we find the formula

$$\int_{T_\delta}^{T_B} \frac{\partial\mu}{\partial T}\frac{S^3T^3}{\rho_n A(T)}\left(\frac{\rho_s}{\rho}\right)^3 dT = -q_B^3 x, \qquad (21)$$

which implicitly expresses the temperature distribution in the helium layer in the large flux case. In the limit case of small fluxes (and respective temperature drops) we return to (11). In the general case, the left side of (21) is integrated numerically. The velocity v_n is then determined from (20). However, it is not now constant but changes so that the product TSv_n remains equal to q_B. Let us note again that we speak about a domain outside the layer l in which everything remains practically without change.

In conclusion, it is interesting to discuss one more question. If q_W, the heat flux from a plate, exceeds q_B, then in what manner is the "excess" heat eliminated? As is known, in ordinary liquids the heat is eliminated either by vapor bubbles or by using convention. But these and other mechanisms are also absent in He II, because its heat conductivity is so large that the existence of bubbles and convective cells is impossible. Any overheated domain being formed is instantly resorbed, where the heat is entrained by second sound [2]. It is reasonable to consider that this mechanism (second sound) which makes bubble or convective cell formation impossible should perform their role during elimination of the "excess" heat flux: $q_W - q_B$. In other words, the flow pattern considered seemingly is "modulated" by a system of chaotic second sound pulses (entropy pulses) that transfer heat to the helium free surface.

NOTATION

ρ, density of helium II; T, temperature; p, pressure; ρ_n, ρ_s, densities; v_n, v_s, normal and superfluid velocity components; S, σ, entropies per unit volume and unit mass; T_B, p_B, temperature and pressure on the free liquid surface; H, depth of submersion; q_B, heat flux transferred through the helium; T, p_δ, temperature and pressure on the interphasal surface; j, mass flux density; η, ζ_1, ζ_2, ζ_3, ζ_4, $\varkappa$, kinetic coefficients [6, 8]; A(T), mutual friction coefficient [2, 5]; α, β, L, vortex ball parameters [5]; Λ_T, Λ_p, combinations of kinetic coefficients (6), (12); l, characteristic length of the near-boundary temperature jump; V, A", x', dimensionless velocity, mutual friction coefficient, and coordinate, respectively; Δp, ΔT, total pressure and temperature drops in the helium layer; p_λ, T_λ, pressure and temperature in the λ transition; $(\partial T/\partial p)_{sat}$, derivative to the equilibrium curve between the liquid and vapor; q_{cr}, critical heat flux density; μ, chemical potential; g, free-fall acceleration; ΔT_{over}, temperature of liquid overheating.

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